NLO SUSY-QCD Corrections to the "EGRET Gamma-Ray Signal"

\[ \tilde{\chi}^0 \tilde{\chi}^0 \rightarrow A^0 \rightarrow b \bar{b} \]

Björn Herrmann
in collaboration with Michael Klasen

LPSC Grenoble

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Outline

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2. Model and Method
3. Analytical Discussion
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Why Corrections at order $\alpha_s$...?

**WMAP mission** has given precise range for Dark Matter relic density in our Universe [astro-ph/0302209]

$$0.0945 \leq \Omega_{CDM} h^2 \leq 0.1287 \quad \text{(at 2$\sigma$)}$$

Supersymmetry provides interesting **Dark Matter candidate** (lightest of the four neutralinos)

**Relic Density calculation** allows to constrain parameter space of supersymmetric models by comparing to WMAP region

$$\frac{dn}{dt} = -3Hn - \langle \sigma_{\text{eff}} v \rangle \left( n^2 - n_{\text{eq}}^2 \right) \quad n \propto \Omega_{CDM} h^2$$

Higher precision in **cross sections** $\langle \sigma_{\text{eff}} v \rangle$ required to obtain better precision in relic density $\Omega_{CDM}$
Several public codes perform relic density calculation within supersymmetric models:

- DarkSUSY [hep-ph/0406204]
- micrOMEGAs [hep-ph/0602198]

At present most processes only implemented at leading order

 Corrections at next-to-leading order are supposed to be important, at least in certain regions of the parameter space...
Why $\tilde{\chi}^0 \tilde{\chi}^0 \rightarrow A^0 \rightarrow b\bar{b}$...?

Annihilation channel into fermion/antifermion pairs is always open

Process claimed to be compatible with the EGRET gamma-ray excess by de Boer et al. for a neutralino mass of 50 - 100 GeV [astro-ph/0408272]

Process supposed to be important in the $A$-funnel parameter region, i.e. at large $\tan \beta$ and large $m_0$ (large $\tan \beta$ favoured by theory...)

Result can be carried over to neutralino-quark scattering cross section by $s$-$t$-crossing (interesting for direct detection...)
Supersymmetric Model

**Phenomenological MSSM** with (only) seven free parameters

\[ m_0, \quad M_2, \quad m_A, \quad \mu, \quad \tan \beta, \quad A_b, \quad A_t \]

**Spectrum calculation:**

- SuSpect [hep-ph/0211331] (particle masses)
- HDecay [hep-ph/9704448] (Higgs decay width)

**Assumptions:**

- **R-parity conservation** (lightest supersymmetric particle stable)
- Dark Matter particle is the **lightest neutralino** \( \tilde{\chi}_1^0 \)
- No squark mixing (i.e. in particular \( m_{\tilde{b}_1} = m_{\tilde{b}_2} \))

Explore the **different regions** of the parameter space
Non-relativistic Limit

Cross sections have to be evaluated in the non-relativistic limit, i.e. expansion in powers of relative velocity

\[ s = 4m^2_\chi \left(1 + \frac{v^2_{\text{rel}}}{4}\right) + O(v^4_{\text{rel}}) \quad \Rightarrow \quad \sigma v_{\text{rel}} = a + bv^2_{\text{rel}} + O(v^4_{\text{rel}}) \]

Thermal averaged cross section needed for relic density calculation

\[ \langle \sigma v_{\text{rel}} \rangle = \int dv_{\text{rel}} f(v_{\text{rel}}, T, m_\chi) \sigma v_{\text{rel}} \]

Boltzmann equation can be integrated numerically to obtain relic density...

\[ \frac{dn}{dt} = -3Hn - \langle \sigma_{\text{eff}} v \rangle (n^2 - n^2_{\text{eq}}) \]
The Born Calculation

Initial state of **Majorana neutralinos** has to be anti-symmetrized


\[
\begin{align*}
\tilde{\chi}_1^0 & \rightarrow A^0 \rightarrow b \bar{b} \\
\tilde{\chi}_1^0 & \rightarrow A^0 \\
\tilde{\chi}_1^0 & \rightarrow A^0
\end{align*}
\]

Cross section important at **large tan} \beta}

\[
\sigma_{LO} \sim 2 \alpha_{ew}^2 N_C \tan^2 \beta \frac{\sqrt{s - 4m_b^2} \sqrt{s - 4m_\chi^2}}{|s - m_A^2 + im_A \Gamma_A|^2}
\]

**Non-relativistic expansion** in powers of relative velocity in agreement with Jungman et al.  \[\text{[hep-ph/9506380]}\]

**Leading order cross section** implemented in public codes

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Virtual QCD Corrections at Next-to-Leading Order

Quark self-energy and vertex correction at order $\alpha_s$, Standard Model and Supersymmetry contributions

Virtual corrections factorize Born cross section

$$\sigma_V = 2 \Lambda_V \sigma_{LO} = 2 (2\Lambda_{bb} + \Lambda_{Abb}) \sigma_{LO}$$

Use dimensional regularization to handle singularities in loop integrals
Handling UV-Singularities in QCD Correction

Unrenormalized virtual correction factor

$$\Lambda_V = \frac{\alpha_S C_F}{2\pi} \left[ \frac{2}{\epsilon_{uv}} - \frac{1 + \beta_b^2}{2\beta_b} \ln \frac{1 - \beta_b}{1 + \beta_b} \left( \frac{1}{\epsilon_{ir}} - \ln \frac{m_b^2}{\mu^2} \right) - 2 \ln \frac{m_b^2}{\mu^2} + \lambda(\beta_b) \right]$$

Counterterms from on-shell renormalization at scale $\mu$

$$\delta Z_m + \delta Z_\psi = \frac{\alpha_S C_F}{2\pi} \left[ -\frac{2}{\epsilon_{uv}} - \frac{1}{\epsilon_{ir}} - 4 + 3 \ln \frac{m_b^2}{\mu^2} \right]$$

UV-singularities vanish in renormalized virtual correction factor, only IR-singularities remain

$$\Lambda_V^{(\text{ren})} = \Lambda_V + (\delta Z_m + \delta Z_\psi)$$

Same procedure for Supersymmetry part, but no IR-singularities...
Handling UV-Singularities in SUSY-QCD Correction

Unrenormalized virtual correction factor

$$\Lambda_V = \frac{\alpha_S C_F}{2\pi} \frac{m_{\tilde{g}} (\mu + A_b \tan \beta)}{\tan \beta} C_0(m_b^2, s, m_b^2, m_b^2, m_{\tilde{g}}^2, m_{\tilde{b}}^2)$$

Counterterms from on-shell renormalization at scale $\mu$

$$\delta Z_m + \delta Z_\psi = \frac{\alpha_S C_F}{2\pi} \left[ \frac{1}{\epsilon_{uv}} - \frac{1}{\epsilon_{uv}} + \frac{A_0^{(\text{fin})}(m_b^2)}{m_b^2} - \frac{A_0^{(\text{fin})}(m_{\tilde{g}}^2)}{m_{\tilde{b}}^2} \right]$$

$$- \frac{(m_b^2 - m_{\tilde{g}}^2)}{m_b^2} + B_0^{(\text{fin})}(m_b^2, m_{\tilde{g}}^2, m_b^2) + \frac{(m_b^2 + m_{\tilde{g}}^2 - m_{\tilde{b}}^2)}{m_b^2} B'_0(m_b^2, m_{\tilde{g}}^2, m_b^2)$$

Renormalized virtual correction factor is free of singularities

$$\Lambda_V^{(\text{ren})} = \Lambda_V + (\delta Z_m + \delta Z_\psi)$$

Correction factor depends on supersymmetric parameters ...
Real Corrections at Next-to-Leading Order

Real gluon emission cancels remaining IR-singularities

\[
\frac{d\sigma_R}{\sigma_{LO}} = \frac{\alpha_S C_F}{2\pi} \left[ \frac{x_1^2 + x_2^2}{(1-x_1)(1-x_2)} - \frac{2m_b^2}{s} \frac{(2-x_1-x_2)^2}{(1-x_1)^2(1-x_2)^2} \right] \frac{dx_1 dx_2}{\beta_b \beta^2_\chi}
\]

Differential real emission cross section

Use Dipole Subtraction Method proposed by Catani et al. to compute virtual and real part numerically [hep-ph/0201036]

\[
\sigma_{NLO} = \left[ \sigma_V + \int d\sigma_{aux} \right]_{\epsilon=0} + \int \left[ d\sigma_R - d\sigma_{aux} \right]_{\epsilon=0}
\]

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Handling IR-Singularities

Use Dipole Subtraction Method proposed by Catani et al. to compute virtual and real part numerically [hep-ph/0201036]

\[ \sigma_{NLO} = [\sigma_V + \int d\sigma_{aux}]_{\epsilon=0} + \int [d\sigma_R - d\sigma_{aux}]_{\epsilon=0} \]

Auxiliary cross section will cancel separately IR-singularities in both virtual and real part of the next-to-leading order cross section

\[ \frac{d\sigma_{aux}}{\sigma_{LO}} = \frac{\alpha_S C_F}{2\pi} \left[ D_{31,2}(x_1, x_2) + D_{32,1}(x_1, x_2) \right] \frac{dx_1 dx_2}{\beta_b \beta^2_{\chi}} \]

\[ \int \frac{d\sigma_{aux}}{2\sigma_{LO}} = \frac{\alpha_S C_F}{2\pi} \left[ \left( 1 + \frac{1 + \beta^2_b}{2 \beta_b} \ln \frac{1 - \beta_b}{1 + \beta_b} \right) \frac{1}{\epsilon_{ir}} - \frac{1}{2} \ln \frac{m_b^2}{\mu^2} + \lambda(\beta_b) \right] \]

Both parts can be computed separately
Final Analytical QCD Result

Cross section at next-to-leading order

\[ \sigma_{\text{NLO}} = 2 \frac{\alpha_S C_F}{2\pi} \Lambda_{\text{NLO}} \sigma_{\text{LO}} \]

where

\[ \Lambda_{\text{NLO}} = \left[ A(\beta_b) - \frac{1}{16\beta_b} \left( 19 + 2\beta_b^2 + 3\beta_b^4 \right) \ln \frac{1 - \beta_b}{1 + \beta_b} + \frac{3}{8} \left( 7 - \beta_b^2 \right) \right] \]

For large \( s \) (\( \beta_b \to 1 \)) correction becomes negative

\[ \Lambda_{\text{NLO}} \approx \left[ \frac{3}{2} \ln \frac{m_b^2}{s} + \frac{9}{4} \right] \]

Resummation of leading logarithmic terms, i.e. use running quark mass


\[ \sigma_{\text{NLO}} = 2 \frac{\alpha_S C_F}{2\pi} \left[ \ln(4m_b^2/\Lambda_{\text{QCD}}) / \ln(s/\Lambda_{\text{QCD}}) \right]^{\frac{24}{33 - 2N_f}} \left[ \Lambda_{\text{NLO}} - \frac{3}{2} \ln \frac{m_b^2}{s} \right] \sigma_{\text{LO}} \]
Motivation

Model and Method

Analytical Discussion

Numerical Results

Conclusion

Born Cross Section

- $\tilde{\chi}_1^0 \tilde{\chi}_1^0 \rightarrow A^0 \rightarrow b \bar{b}$ Born Cross Section

Parameters:

- $m_0 = 1000$ GeV
- $M_2 = 500$ GeV
- $m_A = 1000$ GeV
- $\mu = 300$ GeV
- $A_b = A_t = 3000$ GeV

Process important at large $\tan \beta$
QCD Cross Section at Next-To-Leading Order

NLO corrections enhance cross section by more than 10% at small energies

QCD correction independant from SUSY parameters

Björn Herrmann (LPSC Grenoble)
Dependence on Renormalization Scale

The only scale dependance is in $\alpha_S(\mu)$

Uncertainty of about 30% at "natural scale" $\mu = m_b$, lower uncertainty at higher scale (e.g. $\mu = m_A$). . .
Summary and Outlook

Relic density calculation allows to **constrain supersymmetric models**

Corrections at **next-to-leading order** important for better relic density accuracy

We present **SUSY-QCD corrections** at order $\alpha_S$ to the neutralino annihilation process $\tilde{\chi}^0 \tilde{\chi}^0 \rightarrow A^0 \rightarrow b \bar{b}$ [article in preparation]

The QCD corrections **enhance** the leading order cross section by about 10%

Next steps:

- Evaluate **Supersymmetry corrections** at order $\alpha_S$ numerically
- Evaluate **thermal averaged** cross section
- Evaluate effects on neutralino **relic density**
- ...